Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) dx$$

giving your answer in simplest form.

(4)

$$\int (8x^3 - \frac{3}{2}x^{-\frac{1}{2}} + 5)\, dx = \frac{1}{4} \times 8x^4 + 2 \times -\frac{3}{2}x^{\frac{1}{2}} + 5x + c$$ 1 1 1

$$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$$ 1

Integration : $\int ax^b = \frac{ax^{b+1}}{b+1} + c$

P 6 9 2 0 1 A 0 2 4 8

Question 1 continued

(Total for Question 1 is 4 marks)

2. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

a) $x + 3 = 0 \quad \therefore x = -3$

Substitute $x = -3$ into $f(x)$

$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$

$= -54 + 45 - 6 + 15$ (1)

$f(-3) = 0$

$\therefore (x+3)$ is a factor of $f(x)$ since $f(-3) = 0$ (1)

b) $(x+3)(ax^2 + bx + c) \equiv 2x^3 + 5x^2 + 2x + 15$

$x^3 : \quad a = 2$

$x^2 : \quad 3a + b = 5$

$3(2) + b = 5 \quad \therefore b = -1$ (1)

constant : $3c = 15 \quad \therefore c = 5$ (1)

$\therefore f(x) = (x+3)(2x^2 - x + 5)$

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Question 2 continued

c) $f(x) = 0$: $(x+3)(2x^2 - x + 5) = 0$

if $b^2 - 4ac > 0$, 2 real roots
$b^2 - 4ac = 0$, 1 real root
$b^2 - 4ac < 0$, no real root

$x + 3 = 0$

$b^2 - 4ac = (-1)^2 - 4(2)(5) = -39 < 0$ (1) (1)

$x = -3$

$2x^2 - x + 5 = 0$ has no real solutions

(only solution)

d) $f(x) \longrightarrow f(x-5)$ is a translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$f(x-5) = 0$: $-3 + 5 = 2$

only root from (c)

$\therefore$ $x = 2$ is only real solution to $f(x-5) = 0$ (1)

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

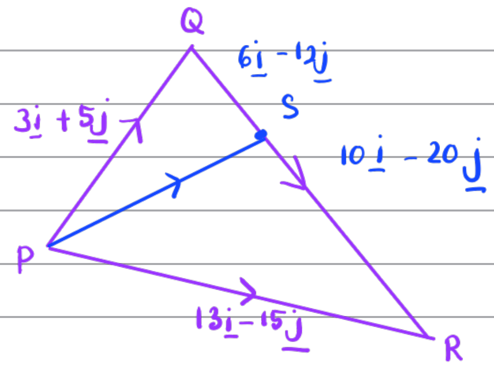
(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS:SR = 3:2$

(c) Find $\overrightarrow{PS}$

(2)



a) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR} = -(3\underline{i} + 5\underline{j}) + (13\underline{i} - 15\underline{j})$ (1)

$= 10\underline{i} - 20\underline{j}$ (1)

b) $|\overrightarrow{QR}| = \sqrt{10^2 + (-20)^2}$ (1)

$= 10\sqrt{5}$ (1)

c) $\overrightarrow{QS} = \frac{3}{5}\overrightarrow{QR} = \frac{3}{5}(10\underline{i} - 20\underline{j}) = 6\underline{i} - 12\underline{j}$ (1)

$\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS} = (3\underline{i} + 5\underline{j}) + (6\underline{i} - 12\underline{j})$ (1)

$= 9\underline{i} - 7\underline{j}$ (1)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

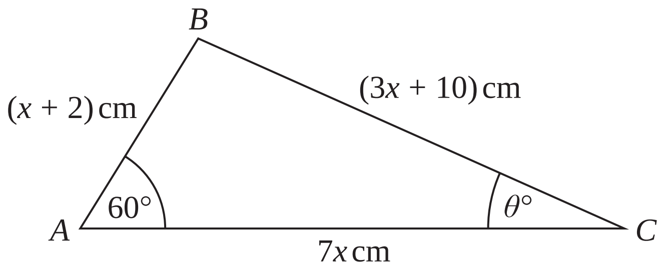


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

a) (i) To get an equation, we can use cosine rule since we have one angle with all 3 sides.

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \angle BAC$

$(3x+10)^2 = (x+2)^2 + (7x)^2 - \cancel{2}(x+2)(7x) \cancel{\cos 60°}$ 1

$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x$ 1

$9x^2 + 60x + 100 = 43x^2 - 10x + 4$

$34x^2 - 70x - 96 = 0$

$\therefore 17x^2 - 35x - 48 = 0$ 1

(ii) $17x^2 - 35x - 48 \equiv (17x + 16)(x - 3) = 0$

$x = -\frac{16}{17}$, $x = 3$ 1   length cannot be negative value

$\therefore$ Since x can only be positive, $x = 3$ is the only solution.

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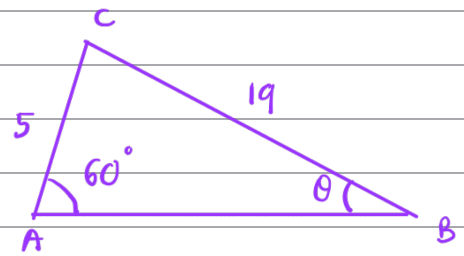
Question 4 continued

b) when $x = 3$,



$AB = (x+2)\text{ cm} = 5 \text{ cm}$

$BC = (3x+10)\text{ cm} = 19 \text{ cm}$

using sin rule to get the angle θ :

$$\frac{\sin\theta}{5\text{ cm}} = \frac{\sin 60°}{19\text{ cm}} \quad (1)$$

$$\sin\theta = \frac{5}{19}\sin 60°$$

$$\theta = \sin^{-1}\frac{5\sqrt{3}}{38} \quad (1)$$

$$= 13.17° \approx 13.2° \ (1\text{ d.p.}) \quad (1)$$

Question 4 continued

Question 4 continued

(Total for Question 4 is 6 marks)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

a) $\log_{10} A = 0.03t + 0.5$

$A = 10^{0.03t + 0.5}$ (1)

$A = 10^{0.03t} \times 10^{0.5}$ (1)

$A = \underbrace{(10^{0.5})}_{p}\underbrace{(10^{0.03})^t}_{q}$ (1)

$\therefore A = 3.162 \times 1.072^t$ (4 s.f.) (1)

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Question 5 continued

b)(i) p represents initial mass of algae (in kg), 0 weeks after the mass of algae was first recorded. 1

(ii) q represents the rate of growth of algae (in kg/week) 1

(c) when $t = 8$, find A

$A = (10^{0.5})(10^{0.03})^{8}$

$= 5.495$

$\therefore A = 5.5$ kg 1 (nearest 0.5 kg)

when $A = 4$, find t

$4 = (10^{0.5})(10^{0.03})^{t}$ 1

$10^{0.03t} = \frac{4}{10^{0.5}} = 1.26\ldots$

$0.03t = \log_{10} 1.26\ldots = 0.102\ldots$

$t = 3.401\ldots$

$= 3.4$ weeks 1

d) The small pond will soon be overcrowded, so it is unlikely for algae to multiply at the same rate. 1

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

1st term 2nd term

a) $\left(3 - \frac{2x}{9}\right)^8 = \binom{8}{0}(3)^8\left(-\frac{2x}{9}\right)^0 + \binom{8}{1}(3)^7\left(-\frac{2x}{9}\right)^1 +$ 1 1

3rd term 4th term

$\binom{8}{2}(3)^6\left(-\frac{2x}{9}\right)^2 + \binom{8}{3}(3)^5\left(-\frac{2x}{9}\right)^3 + \ldots$ 1

$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \ldots$ 1

b) $\left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$

$\left(\frac{x}{2x} - \frac{1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$

$\left(\frac{1}{2} - \frac{1}{2x}\right)\left(6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \ldots\right)$

coefficient of x^2 : $\left(\frac{1}{2} \times 1008\right) + \left(-\frac{1}{2} \times -\frac{448}{3}\right)$ 1

$= 504 + \frac{224}{3}$

$= \frac{1736}{3}$ 1

P 6 9 2 0 1 A 0 1 8 4 8

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

a) $9x - x^3 \equiv x(9 - x^2)$ ①

$\equiv x(3+x)(3-x)$ ①

b) $y = 9x - x^3$

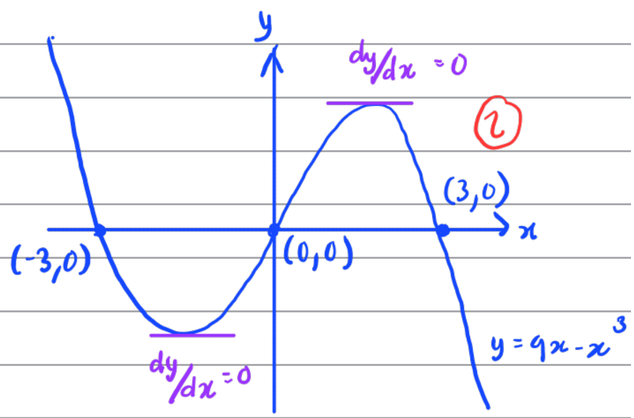
when $x = 0$, $y = 9(0) - (0)^3 = 0$

when $y = 0$, $0 = 9x - x^3$

$0 = x(3+x)(3-x)$

$x = -3, 0, 3$

Curve C :



②

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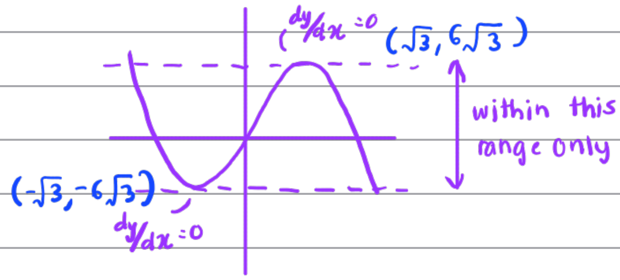
Question 7 continued

c) For line ℓ to intersect at 3 points of curve C, the intersection points can only be within 2 turning points of the curve

Turning points : $\frac{dy}{dx} = 0$



$\frac{dy}{dx} = 9 - 3x^2$

$\therefore 9 - 3x^2 = 0$ (1)

$x^2 = 3$

$x = \pm\sqrt{3}$

when $x = \sqrt{3}$, $y = 9\sqrt{3} - (\sqrt{3})^3 = 6\sqrt{3}$ (1)

$x = -\sqrt{3}$, $y = 9(-\sqrt{3}) - (-\sqrt{3})^3 = -6\sqrt{3}$

$\therefore \{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$ (1)

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

a) $t = 0,\ p = 2.2 \quad : \quad 2.2 = k + 1.4e^{0}$

$2.2 = k + 1.4$

$\therefore\ k = 0.8$ (1)

$\therefore\ p = 0.8 + 1.4e^{-0.5t}$

b) $p = 1 \quad : \quad 1 = 0.8 + 1.4e^{-0.5t}$

$1.4e^{-0.5t} = 0.2$ (1)

$e^{-0.5t} = \frac{1}{7}$

$-0.5t = \ln\left(\frac{1}{7}\right)$ (1)

$t = -2\ln\left(\frac{1}{7}\right)$

$\therefore\ t = 3.9$ minutes (1 d.p.) (1)

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Question 8 continued

c) $\frac{dP}{dt} = -0.7e^{-0.5t}$ (1)

when $t = 2$: $\frac{dP}{dt} = -0.7e^{-0.5(2)}$ (1) $= -0.2575...$

$\therefore$ decreasing at a rate of 0.258 kg/cm^2 (3 s.f.) (1)

(Total for Question 8 is 6 marks)

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

a) (i) $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9$

$= p - 2$ 1

(ii) $\log_3 \sqrt{x} = \log_3 x^{\frac{1}{2}}$

$= \frac{1}{2}\log_3 x$

$= \frac{1}{2}p$ 1

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Question 9 continued

use value from (a)

b) $2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$ (1)

$2(p-2) + 3\left(\frac{1}{2}p\right) = -11$

$2p - 4 + \frac{3}{2}p = -11$

$4p - 8 + 3p = -22$

$7p = -14$

$p = -2$ (1)

$\Rightarrow p = \log_3 x$

$\log_3 x = -2$ (1)

$x = 3^{-2} = \frac{1}{9}$ (1)

(Total for Question 9 is 6 marks)

10.

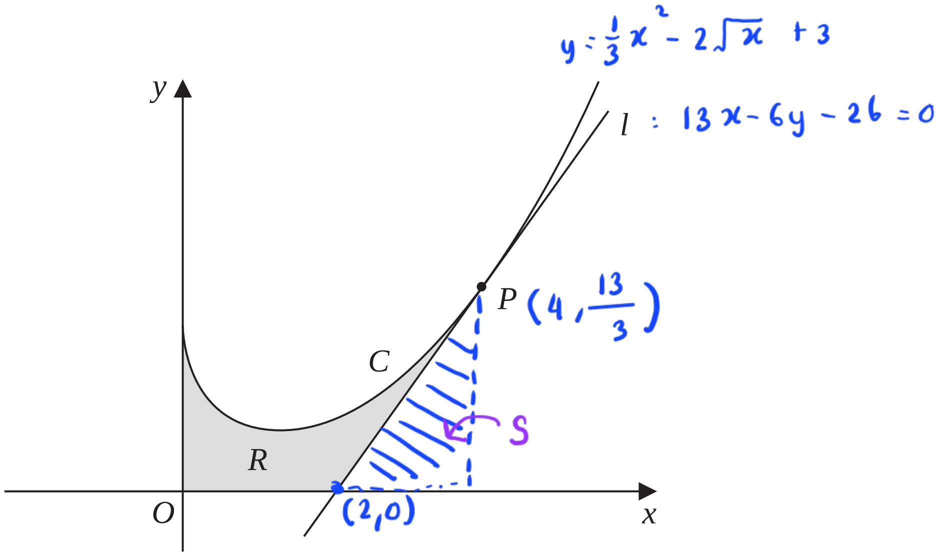

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

a) $y = \frac{1}{3}x^2 - 2x^{\frac{1}{2}} + 3$

$\frac{dy}{dx} = \frac{2}{3}x - x^{-\frac{1}{2}}$ 1

when $x = 4$, $y = \frac{1}{3} \times 4^2 - 2 \times 4^{\frac{1}{2}} + 3 = \frac{13}{3}$ $\therefore P\left(4, \frac{13}{3}\right)$

$\frac{dy}{dx} = \frac{2}{3} \times 4 - 4^{-\frac{1}{2}} = \frac{13}{6}$ 1 (gradient of tangent)

Question 10 continued

Finding equation of line l :

$P\left(4, \frac{13}{3}\right)$: $y - \frac{13}{3} = \frac{13}{6}(x-4)$

$6y - 2 \times 13 = 13(x-4)$

$6y - 26 = 13x - 52$

$\therefore \; l : 13x - 6y - 26 = 0$ (1)

b) Finding the x-intercept of line l :

when $y = 0$, $13x - 0 - 26 = 0$

$13x = 26 \longrightarrow \therefore x = 2$ (1)

Finding area under curve :

$$\int_0^4 \left(\frac{1}{3}x^2 - 2x^{\frac{1}{2}} + 3\right) dx$$

$$= \left[\frac{1}{9}x^3 - \frac{4}{3}x^{\frac{3}{2}} + 3x\right]_0^4 \quad (2)$$

$$= \left\{\frac{1}{9}(4)^3 - \frac{4}{3}(4)^{\frac{3}{2}} + 3(4)\right\} - \{0 - 0 + 0\}$$

$$= \frac{76}{9}$$

Question 10 continued

Finding area of triangle S :

$\frac{1}{2} \times 2 \times \frac{13}{3}$ (1)

$= \frac{13}{3}$

$\therefore$ Area of R = area under curve − area of triangle S

$= \frac{76}{9} - \frac{13}{3}$

$= \frac{37}{9}$ (1)

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Question 10 continued

(Total for Question 10 is 10 marks)

11.

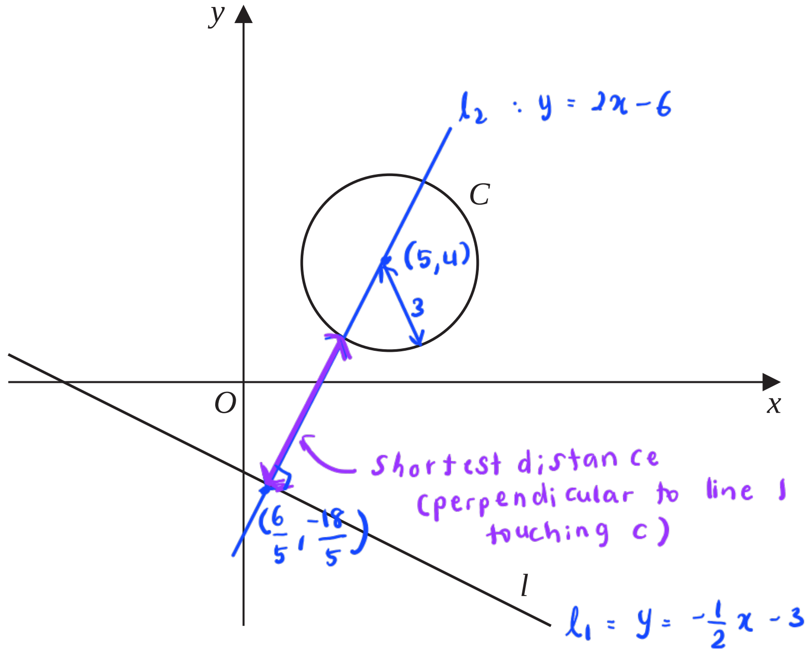

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

$(x-a)^2 + (y-b)^2 = r^2$

a) (i) $x^2 + y^2 - 10x - 8y + 32 = 0$

$(x-5)^2 - 5^2 + (y-4)^2 - 4^2 + 32 = 0$

$(x-5)^2 + (y-4)^2 = 9$

$(x-5)^2 + (y-4)^2 = 3^2$ 1

$\therefore$ centre $(5, 4)$ 1

(ii) radius $= 3$ 1

Question 11 continued

b) $l_1 = 2y + x + 6 = 0$

$2y = -x - 6$

$y = -\frac{1}{2}x - 3$ $\therefore$ gradient of l_1 is $-\frac{1}{2}$

$\therefore$ gradient of l_2 is $\frac{-1}{-\frac{1}{2}} = 2$ (1)

Finding equation of line l_2 :

Known $(5,4)$ from centre of circle :

$y - 4 = 2(x-5)$ (1)

$\therefore l_2 : y = 2x - 6$

Finding intersect point of l_1 and l_2 :

$l_1 : y = -\frac{1}{2}x - 3$ — (1)

$l_2 : y = 2x - 6$ — (2)

subs (2) into (1)

$-\frac{1}{2}x - 3 = 2x - 6$

$-x - 6 = 4x - 12$

$5x = 6 \rightarrow \therefore x = \frac{6}{5}$

$y = 2\left(\frac{6}{5}\right) - 6 \longrightarrow y = -\frac{18}{5}$ (1)

Question 11 continued

Finding distance from l_1 to centre of C :

$$= \sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 - \left(-\frac{18}{5}\right)\right)^2} \quad (1)$$

$$= \frac{19\sqrt{5}}{5}$$

Finding distance from l_1 to C :

radius of C

$$\frac{19\sqrt{5}}{5} - 3 \quad (1)$$

$$= 5.50$$

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Question 11 continued

(Total for Question 11 is 8 marks)

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is $355\,\text{cm}^3$

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

a) $\pi r^2 h = 355 \quad \therefore h = \frac{355}{\pi r^2}$ (1)

cost of base $= 0.04\,\pi r^2$

cost of side $= 0.04 \times 2\pi r h = 0.04 \times 2\pi r \left(\frac{355}{\pi r^2}\right) = \frac{28.4}{r}$

cost of top $= 0.09\,\pi r^2$

total cost, $C = 0.04\,\pi r^2 + \frac{28.4}{r} + 0.09\,\pi r^2$ (1)

$C = 0.13\,\pi r^2 + \frac{28.4}{r}$ (1)

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Question 12 continued

b) C is minimum when $\frac{dC}{dr} = 0$

$$\frac{dC}{dr} = 0.26\pi r - \frac{28.4}{r^2} \quad (1)$$

$$\frac{dC}{dr} = 0, \quad 0.26\pi r - \frac{28.4}{r^2} = 0$$

$$r^3 - \frac{28.4}{0.26\pi} = 0$$

$$r^3 = \frac{28.4}{0.26\pi} \quad (1)$$

$$r = 3.26 \ (3\text{ s.f.}) \quad (1)$$

c) $$\frac{d^2C}{dr^2} = 0.26\pi + \frac{56.8}{r^3} \quad (1)$$

when $r = 3.26$, $0.26\pi + \frac{56.8}{(3.26)^3}$

$= 2.45$ which is > 0. Hence, cost is minimised.
(1)

d) when $r = 3.26$, $C = 0.13\pi(3.26)^2 + \frac{28.4}{3.26}$ (1)

$= 13$ (1)

$\therefore$ The minimum cost is 13p.

Question 12 continued

Question 12 continued

(Total for Question 12 is 12 marks)

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

LHS :

$$\frac{1}{\cos\theta} + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

1 1

method to get $(1-\sin^2\theta)$ at numerator

$$= \frac{1-\sin^2\theta}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta\,(1-\sin\theta)} \qquad 1-\sin^2\theta = \cos^2\theta$$

$$= \frac{\cos\theta}{1-\sin\theta} \equiv \text{RHS}$$

1

Question 13 continued

b) $\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$ where $0° < x < 90°$

for $2x$: $0° < 2x < 180°$

from a) $\frac{1}{\cos\theta} + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$

$\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ (1)

$\cos 2x = 3\cos 2x(1-\sin 2x)$ (1)

$\cos 2x - 3\cos 2x(1-\sin 2x) = 0$

$\cos 2x(1-3(1-\sin 2x)) = 0$

$\cos 2x(3\sin 2x - 2) = 0$

$\cos 2x \neq 0$, $3\sin 2x - 2 = 0$

$\sin 2x = \frac{2}{3}$ (1)

$2x = 41.81\ldots, 138.18\ldots$

$x = 20.9°, 69.1°$ (1 d.p.)
(1) (1)

Question 13 continued

Question 13 continued

(Total for Question 13 is 8 marks)

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

(i) claim : if $x^2 > 9$, then $x > 3$

if $x = -4$: $(-4)^2 = 16 > 9$ and $(-4) < 3$,

1

then the statement is false

$x^2 > 9$

$= x^2 - 9 > 0$

$= (x+3)(x-3) > 0$

$\therefore x > 3$, $x < -3$ (statement is false)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 14 continued

(ii) $n^3 + 3n^2 + 2n = n(n^2 + 3n + 2)$ (1)

$= n(n+1)(n+2)$, which is the product of three consecutive integers. (1)

Examples of consecutive integers : 7, 8, 9

16, 17, 18

23, 24, 25

(1) one of the numbers is divisible by 3

(2) at least one of them will be divisible by 2

As $n(n+1)(n+2)$ is a multiple of 2 and a multiple of 3, it must be a multiple of 6.

(1)

$\therefore$ So, $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n.

Question 14 continued

(Total for Question 14 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS